

Radiation pressure as a source of decoherence

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We consider the interaction of an harmonic oscillator with the quantum field via radiation pressure. We show that a “Schrödinger cat” state decoheres in a time scale that depends on the degree of “classicality” of the state components, and which may be much shorter than the relaxation time scale associated to the dynamical Casimir effect. We also show that decoherence is a consequence of the entanglement between the quantum states of the oscillator and field two-photon states. With the help of the fluctuation-dissipation theorem, we derive a relation between decoherence and damping rates valid for arbitrary values of the temperature of the field. Coherent states are selected by the interaction as pointer states.

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I. INTRODUCTION

Superposition states have an important role in the formalism of quantum mechanics. However, they are in flagrant contradiction with our classical world when the components correspond to macroscopically distinguishable states. The reason why these states are not encountered classically is decoherence, a process by which the interaction between the degrees of freedom of the system in question with any other degrees of freedom, either internal or external (the so-called environment), leads to a suppression of the coherence between the components of the superposition [1]. Even if this coupling is very weak, the decoherence rate may be huge, resulting in a very fast decay of these “weird” states and in the emergence of the classical world. Recent developments in technology now allow one to study in real time the process of decoherence in the lab. For example, over the past several years techniques have been developed to generate mesoscopic superpositions of motional states of trapped ions [2], and of photon states in cavity quantum electrodynamics [3]. In these cases decoherence due to the coupling with the ambient reservoirs was observed, confirming the expectation that the decoherence rate is faster, the larger and more separated the state components are [4]. Recently another experiment has succeeded in “engineering” the environment in the context of trapped ions, studying scaling laws of decoherence theory for a variety of reservoirs in a wide range of parameters [5].

Usually, decoherence is analyzed in the framework of heuristic models that describe phenomenological dissipation (the reservoir is taken to be a collection of harmonic oscillators, coupled linearly to the position operator of the system [6,7]). In this paper, we consider instead an *ab initio* model for decoherence of a particle in a harmonic potential, scattering the radiation field (at temperature T), which then plays the role of the reservoir. Starting from first principles, we show that the resulting radiation pressure coupling with the

field transforms an initial pure superposition state of the particle into a statistical mixture.

Of special relevance is the limit $T=0$. In this case, the reservoir is the quantum vacuum field, which dissipates the mechanical energy of the oscillating particle (or “mirror”). This effect is associated to the emission of pairs of photons, the so-called dynamical Casimir effect. Much work has been done on quantum radiation from moving mirrors [8]. Important properties like the spectrum of the emitted radiation [9], the time evolution of the energy-momentum tensor [10], the total radiated intensity and the dissipative radiation pressure on the particle (radiation reaction force corresponding to the photon emission effect) [11,12] have been considered. Here we focus our attention on the particle as the system of interest, and show that decoherence is a consequence of the entanglement between particle and field two-photon states. This result has fundamental implications, for it shows that any particle not transparent to the radiation is unavoidably under the action of decoherence through the radiation pressure coupling with vacuum fluctuations.

The zero temperature limit was briefly discussed in our previous letter [13]. This paper presents results for finite values of temperature, as well as a detailed discussion of the case $T=0$. The formalism relies on the one-dimensional (1D) scalar model for the field, but extensions to a 3D electromagnetic field are also discussed, allowing us to address the question of orders of magnitude. The paper is organized as follows. In Sec. II we start from the Hamiltonian model for the radiation pressure coupling, and then derive a master equation for the particle. In Sec. III we discuss how the environment selects a preferred basis in the particle’s Hilbert space, the pointer basis. In Section IV we derive a general relation between decoherence and damping rates at arbitrary temperature by means of the fluctuation-dissipation theorem. The zero- and high-temperature limits, including extensions to the 3D electromagnetic model, are discussed in Secs. V and VI. Section VII contains our conclusions. Finally, in the appendix an alternative, simpler derivation of the decoherence rate is given, which is based on the entanglement between the particle and two-photon states.

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II. MODEL

Most treatments of the dynamical Casimir effect are based on the assumption that the mirror follows a prescribed trajectory, thus neglecting the recoil effect. However, in this paper we want to focus on the mirror as a dynamical quantum system, hence the need to tackle the full mirror-plus-field dynamics. This has already been addressed in the framework of linear-response theory in order to calculate the fluctuations of the position of a dispersive mirror driven by vacuum radiation pressure [14], and related calculations have been performed in Refs. [15–17] to derive mass corrections caused by the interaction with the field.

We consider a nonrelativistic partially reflecting mirror of mass M (with position q and momentum p) in a harmonic potential of frequency ω_0 , and under the action of radiation pressure. We take a scalar field in 1+1 dimensions, which mimics the electromagnetic field modes that propagate along the direction perpendicular to the plane of the mirror. Extensions to the real 3+1 case are analyzed in Secs. V and VI. We neglect third- and higher-order terms in v/c , where v is the mirror's velocity (we set $c=1$ hereafter, except when an explicit evaluation of orders of magnitude is required). Our starting point is the Hamiltonian formalism developed in Refs. [16] and [17] (although these references consider a free mirror, the extension to the harmonic oscillator is straightforward). The total Hamiltonian is

$$H = H_M + H_F + H_{\text{int}}, \quad (1)$$

where

$$H_M = \frac{p^2}{2M} + \frac{M\omega_0^2}{2} q^2 \quad (2)$$

is the harmonic oscillator Hamiltonian for the mirror, and

$$H_F = \int \frac{dx}{2} [\Pi^2 + (\partial_x \phi)^2] + \Omega \phi^2(x=0) \quad (3)$$

is the free Hamiltonian for the field ϕ and its momentum canonically conjugated $\Pi = \partial_t \phi$. The second term on the right-hand side of Eq. (3) is associated to the boundary condition of a partially reflecting mirror at rest at $x=0$. In the context of the plasma sheet model of Ref. [16], it corresponds to the kinetic energy of the plasma charged particles. The coupling constant Ω plays the role of a transparency frequency, since from Eq. (3) one derives the boundary condition

$$\partial_x \phi(0^+) - \partial_x \phi(0^-) = 2\Omega \phi(0)$$

(ϕ is continuous at $x=0$), which yields a frequency-dependent reflection amplitude [14,16]:

$$R(\omega) = -i \frac{\Omega}{\omega + i\Omega}. \quad (4)$$

Finally, the interaction Hamiltonian is given by

$$H_{\text{int}} = -\frac{p\mathcal{P}}{M} + \frac{\mathcal{P}^2}{2M} - \frac{1}{2}\Omega \phi^2(x=0) \frac{p^2}{M^2}, \quad (5)$$

where $\mathcal{P} = -\int dx \partial_x \phi \partial_t \phi$ is the field momentum operator. H_{int} describes, to second order in v/c , the modification of the boundary condition for the field due to the motion of the mirror, which in its turn is affected by the field radiation pressure. Thus, it provides a coupling between the harmonic oscillator and the field, to be treated within perturbation theory. The small perturbation parameter is v/c , and not the transparency frequency Ω , which may be arbitrarily large. The first term in Eq. (5) is responsible for the effect of decoherence to be discussed here. It also accounts for the effects of emission of photons, dissipation of the mirror's energy, and part of the mass correction.

We calculate the density matrix $\tilde{\rho}(t)$ of the combined mirror-plus-field system using second-order perturbation theory, and trace over the field operators to derive the master equation for the mirror's density matrix $\rho(t)$ [18]. We assume that at $t=0$ the mirror and field are not correlated: $\tilde{\rho}(0) = \rho(0) \otimes \rho_F$, where ρ_F is the density matrix of the field (assumed to be in some steady state; later in this section we take a thermal equilibrium state). We find

$$\begin{aligned} i\hbar \dot{\rho}(t) = & [H_M, \rho(t)] - \Omega \frac{\langle \phi^2(0) \rangle}{2M^2} [p^2, \rho(t)] \\ & - \frac{i}{2\hbar M^2} \int_0^t dt' ([p, [p^I(-t'), \rho(t)]] \sigma(t') \\ & + [p, \{p^I(-t'), \rho(t)\}] \xi(t')), \end{aligned} \quad (6)$$

where the superscript I indicates the operators to be taken in the interaction picture. The second term on the right-hand side of Eq. (6) is the contribution in first order of perturbation theory of the p^2 term in the interaction Hamiltonian [see Eq. (5)]. It corresponds to a (cutoff dependent) mass correction given by

$$\Delta M_1 = \Omega \langle \phi^2(0) \rangle, \quad (7)$$

as already found in Refs. [14] and [16]. The (anti-)symmetric second-order correlation function (ξ) σ is defined as

$$\sigma(t) = C(t) + C(-t), \quad (8)$$

$$\xi(t) = C(t) - C(-t), \quad (9)$$

with

$$C(t) = \langle \mathcal{P}^I(t) \mathcal{P}^I(0) \rangle - \langle \mathcal{P} \rangle^2. \quad (10)$$

When computing the correlation functions, we take the unperturbed field, which corresponds to the static boundary condition (eigenfunctions of H_F).

Replacing the free evolution for $p^I(-t')$ in Eq. (6) yields

$$i\hbar\dot{\rho} = \left[H_M - \frac{\Delta M(t)}{M} \frac{p^2}{2M}, \rho \right] - \Gamma(t)[p, \{q, \rho\}] - \frac{i}{\hbar} D_1(t)[p, [p, \rho]] - \frac{i}{\hbar} D_2(t)[p, [q, \rho]]. \quad (11)$$

The total mass correction is $\Delta M = \Delta M_1 + \Delta M_2$, where ΔM_2 , as well as the remaining coefficients in Eq. (11), originate from the first term on the right-hand-side of Eq. (5), taken in second-order perturbation theory. Their meanings are best understood when writing the Fokker-Planck equation for the Wigner function $W(x, p, t)$:

$$\begin{aligned} \partial_t W = & -(1 - \Delta M/M) \frac{p}{M} \partial_x W + M \omega_0^2 x \partial_p W + 2\Gamma \partial_x(xW) \\ & + D_1 \frac{\partial^2}{\partial x^2} W - D_2 \frac{\partial^2}{\partial x \partial p} W. \end{aligned} \quad (12)$$

ΔM_2 and the damping coefficient Γ are calculated from the anti-symmetric correlation function:

$$\Delta M_2(t) = \frac{i}{\hbar} \int_0^t dt' \cos(\omega_0 t') \xi(t'), \quad (13)$$

$$\Gamma(t) = \frac{i\omega_0}{2M\hbar} \int_0^t dt' \sin(\omega_0 t') \xi(t'); \quad (14)$$

whereas the diffusion coefficients are associated to the symmetric correlation function:

$$D_1(t) = \frac{1}{2M^2} \int_0^t dt' \cos(\omega_0 t') \sigma(t'), \quad (15)$$

$$D_2(t) = \frac{\omega_0}{2M} \int_0^t dt' \sin(\omega_0 t') \sigma(t'). \quad (16)$$

We assume the field to be in a thermal state (temperature T), and take the following strategy to calculate the momentum correlation functions. The time derivative of the field momentum is minus the radiation pressure force on the mirror [16]:

$$\frac{d\mathcal{P}^I}{dt} = 2\Omega \phi(0, t) \bar{\partial}_x \phi(0, t), \quad (17)$$

where $\bar{\partial}_x \phi(0, t) = [\partial_x \phi(0^+) + \partial_x \phi(0^-)]/2$. Using Eq. (17), we calculate $C(t)$ by integrating the correlation function of the field calculated at $x=0$:

$$\begin{aligned} C(t) = & (2\Omega)^2 \int_{-\infty}^t dt_1 \int_{-\infty}^0 dt_2 [\langle \phi(0, t_1) \bar{\partial}_x \phi(0, t_1) \\ & \times \phi(0, t_2) \bar{\partial}_x \phi(0, t_2) \rangle \\ & - \langle \phi(0, t_1) \bar{\partial}_x \phi(0, t_1) \rangle \langle \phi(0, t_2) \bar{\partial}_x \phi(0, t_2) \rangle] \end{aligned} \quad (18)$$

The equal-time second-order correlation function in Eq. (18) corresponds to the force on the static (single) mirror. It vanishes since the vacuum radiation pressures exerted on each side of the mirror are in perfect equilibrium. On the other hand, the fourth-order correlation function may be expressed as a sum of second-order correlation functions (with the fields taken at different times), which are calculated with the help of the normal mode expansion for the field. They are directly connected to the average number of photons $n_\omega = 1/[\exp(\hbar\omega/T) - 1]$ in the mode of frequency ω at temperature T (we take the Boltzmann constant $k_B = 1$). It is useful to write the result in the Fourier domain, the Fourier transform of the antisymmetric correlation function $\xi(t)$ being defined as

$$\xi[\omega] = \int dt \exp(i\omega t) \xi(t). \quad (19)$$

Equations (9) and (18) yield

$$\xi[\omega] = \xi^0[\omega] + \xi^T[\omega], \quad (20)$$

where

$$\xi^0[\omega] = (2/\pi) \hbar^2 \Omega \zeta(\omega/\Omega) \quad (21)$$

with

$$\zeta(u) = \ln(1 + u^2)/(2u) + (\arctan u)/u^2 - 1/u \quad (22)$$

represents the correlation function at $T=0$ (vacuum fluctuations), whereas

$$\xi^T[\omega] = \frac{2\hbar^2 \Omega^2}{\pi \omega^2} \int_0^\infty d\omega' \frac{\omega'}{\omega'^2 + \Omega^2} [G(\omega, \omega') - G(-\omega, \omega')] \quad (23)$$

with

$$G(\omega, \omega') = |\omega' - \omega| (n_{|\omega' - \omega|} - \epsilon(\omega' - \omega) n_{\omega'}) \quad (24)$$

represents the thermal fluctuations (ϵ is the sign function).

Symmetric and antisymmetric correlation functions for a system in thermal equilibrium are related in a very general way [25,26]:

$$\sigma[\omega] = \frac{\xi[\omega]}{\tanh\left(\frac{\hbar\omega}{2T}\right)}. \quad (25)$$

According to Eqs. (14) and (15), this result provides a general relation between diffusion and damping, in the spirit of the fluctuation-dissipation theorem. This relation is particularly simple for the asymptotic values of the coefficients $\Gamma(t)$ and $D_1(t)$ at $t \rightarrow \infty$. Since the integrands in Eqs. (14) and (15) are even functions of time, we may extend the integration range to $-\infty$, yielding

$$\Gamma = \frac{\omega_0}{4M\hbar} \xi[\omega_0] \quad (26)$$

and

$$D_1 = \frac{1}{4M^2} \sigma[\omega_0]. \quad (27)$$

Thus, the asymptotic values of Γ and D_1 are directly connected to the fluctuations at the mechanical frequency ω_0 , allowing us to derive, from Eq. (25), a simple and general relation between these two coefficients. On the other hand, no such simple connection exists for the remaining time-dependent coefficients, ΔM_2 and D_2 , whose asymptotic values result from the joint contribution of the whole spectrum of fluctuations [13].

Combining Eqs. (25)–(27), we find

$$D_1 = \frac{\hbar}{M\omega_0} \frac{\Gamma}{\tanh(\hbar\omega_0/2T)}, \quad (28)$$

a clear manifestation of the fluctuation-dissipation theorem. According to Eq. (28), the temperature dependence of the diffusion coefficient is determined, apart from the T dependence of the damping coefficient Γ (to be discussed later), by the relative importance of the thermal fluctuations (and their corresponding energy T) with respect to quantum fluctuations (and their corresponding zero-point energy $\hbar\omega_0/2$). In the high-temperature limit, $T \gg \hbar\omega_0/2$, Eq. (28) yields $D_1 = 2T\Gamma/(M\omega_0^2)$. In the theory of Brownian motion, Γ is a T independent phenomenological constant, and hence the diffusion coefficient is a linear function of temperature in this limit. Here, however, Γ has an explicit temperature dependence, to be analyzed in Sec. V.

From Eq. (28), we shall derive a relation between decoherence and damping time scales, valid for any temperature T . Before considering a specific superposition state, however, we discuss, in the next section, the degree of sensitivity of different states in the Hilbert space to the action of decoherence. We also analyze in more detail the precise meaning of $t \rightarrow \infty$ (in the particular case of $T=0$), in order to know how fast the coefficients approach their asymptotic values. From Eqs. (14) and (15) alone it may be shown, in a general way, that a *sufficient* condition is $t \gg 1/\omega_0$, but in some cases the convergence may be much faster.

III. POINTER STATES

Different criteria have been introduced in the literature in order to find out the states in the Hilbert space that are most robust under the interaction with the environment and behave more classically [19–21]. Here we shall follow the one introduced by Zurek, the so-called “predictability sieve.” The idea is to take every possible state of the Hilbert space, calculate its entropy, and order the states in a tower according to increasing entropy. The most classical states are those that lie at the bottom of that tower, and correspond to the most predictable ones. For these “pointer” states, information loss due to the interaction with the environment is minimal. This philosophy is put in quantitative terms by minimizing the linear entropy of the system,

$$s[\rho] \equiv 1 - \text{tr} \rho^2, \quad (29)$$

which is zero for a pure state and greater than zero for a statistical mixture. In general this is a difficult problem because complicated entanglement between system and environment develops on account of their mutual interaction. So far, results have been successfully derived assuming that the initial state of the system is pure. Here we follow the same approach, and calculate the rate of entropy increase starting from the master equation (11). We assume that the state is nearly pure at time t to find

$$\dot{s}(t) = 2\Gamma(t)[s(t) - 1] + \frac{4D_1(t)}{\hbar^2} \Delta p^2 + \frac{2D_2(t)}{\hbar^2} \sigma_{q,p}, \quad (30)$$

where $\Delta p^2 \equiv \langle p^2 \rangle - \langle p \rangle^2$ is the momentum dispersion and $\sigma_{q,p} \equiv \langle \{q, p\} \rangle - 2\langle p \rangle \langle q \rangle$. Here $\langle \dots \rangle = \text{tr}(\dots \rho)$, and all operators are evaluated at the same time t . The first term in Eq. (30) leads to a decrease of entropy $s(t) = 1 - \exp[2\int_0^t \Gamma(t') dt']$, hence damping tries to localize the state competing against diffusion. This decrease is independent of the initial state of the system, and therefore is irrelevant for determining the pointer states.

We assume that the typical decoherence time scale is much larger than the period of free oscillation $2\pi/\omega_0$, so that we may integrate Eq. (30) to find the entropy at an intermediate time $\tau = n 2\pi/\omega_0$. We take $n \gg 1$, allowing us to replace the time dependent coefficients by their constant asymptotic values, but assume that τ is much shorter than the decoherence time scale, in order to be consistent with the small-entropy approximation underlying Eq. (30). Moreover, in this weak-coupling limit, we may take the free evolution (corresponding to the harmonic oscillator Hamiltonian H_M) for the mirror's operators in Eq. (30). The correlation function $\sigma_{q,p}$ oscillates around zero, and then does not contribute to $s(\tau)$, whereas the free evolution of $\Delta p^2(t)$ mixes up position and momentum fluctuations, yielding

$$s(\tau) = 2\tau \frac{D_1}{\hbar^2} [(\Delta p)_0^2 + (M\omega_0)^2 (\Delta q)_0^2 - M\hbar\omega_0], \quad (31)$$

where $(\Delta p)_0^2$ and $(\Delta q)_0^2$ represent the dispersions for the initial state. From Eq. (31), we find that the minimum entropy given the constraint $\Delta q \Delta p \geq \hbar/2$ is for $\Delta q^2 = \hbar/(2M\omega_0)$, $\Delta p^2 = M\hbar\omega_0/2$. Hence, as in the Caldeira-Leggett model, and for any temperature of the field, the pointer basis consists of coherent states [20]. In this weak-coupling approximation, the minimum value corresponds to $s(\tau) = 0$, hence the increase of entropy of a coherent state is a higher-order effect.

The crucial approximation in the derivation of Eq. (31) from Eq. (30) is the replacement of the time dependent coefficients by their finite, constant asymptotic values. It is instructive to analyze in more detail the behavior of the coefficients and its connection with entropy production. As an example, we take $T=0$, and consider first the perfectly reflecting limit, which corresponds to $\omega_0 \ll \Omega$, for in this case the relevant field modes have frequencies much smaller than

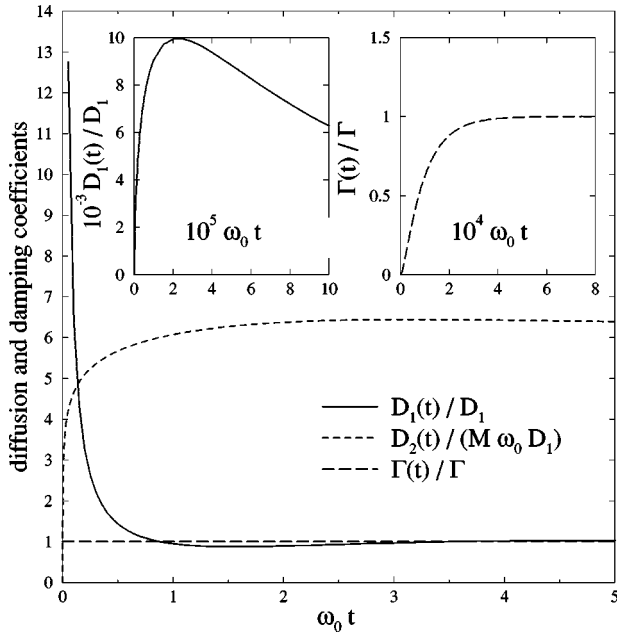


FIG. 1. Diffusion and damping coefficients for zero temperature as functions of time in the perfectly reflecting limit, $\omega_0/\Omega = 10^{-4} \ll 1$. Here $D_1 = \hbar^2 \omega_0 / 12\pi M^2$ and $\Gamma = \hbar \omega_0^2 / 12\pi M$ are the asymptotic limits of $D_1(t)$ and $\Gamma(t)$. The insets show the behavior of these two time-dependent coefficients for short times.

the mirror's transparency frequency. In Fig. 1 we plot the diffusion and damping coefficients as functions of $\omega_0 t$ for $\omega_0/\Omega = 10^{-4}$ and $T=0$. The damping coefficient Γ approaches its asymptotic value very fast, for $t \sim 1/\Omega$, whereas $D_1(t)$ develops an initial jolt for times of the order of Ω^{-1} and then decreases to the asymptotic value $(D_1)_{\text{perf}} = \hbar^2 \omega_0 / (12\pi M^2)$ for $t \sim 1/\omega_0$. When we integrate Eq. (30) over many periods of oscillation, the contribution to the entropy of the initial jolt is negligible, allowing us to replace D_1 by its asymptotic value.

In Ref. [27], it was shown that no net entropy is produced for the Caldeira–Leggett model with an adiabatic environment, since all the time-dependent coefficients are oscillating functions around a zero mean. At first sight, the same would happen in our model when $\omega_0 \gg \Omega$, for in this case the dominant field frequencies are slow with respect to the mirror's translational time scale. However, as discussed in Sec. V, the spectral density $\xi^0(\omega)$ decays too slowly for $\omega \gg \Omega$, and as a consequence field frequencies of the order of ω_0 provide a significant contribution even in this limit. Thus, one cannot ascribe a frequency cut off to the environment such that the typical frequency of the system ω_0 is much greater than the maximum frequency of the environment. Therefore, the vacuum field does not behave adiabatically in the sense of Ref. [27]. In our case instead, the diffusion coefficients oscillate around a nonzero value, leading to a net entropy increase. In Fig. 2, we plot the diffusion and damping coefficients as functions of $\omega_0 t$ for $\omega_0/\Omega = 10^4$ and $T=0$. They oscillate around their asymptotic values with (angular) frequency ω_0 and with an amplitude of oscillation that decays in a time $t \sim 1/\Omega \gg 1/\omega_0$. The oscillatory terms do not contribute to the entropy increase when we average over many

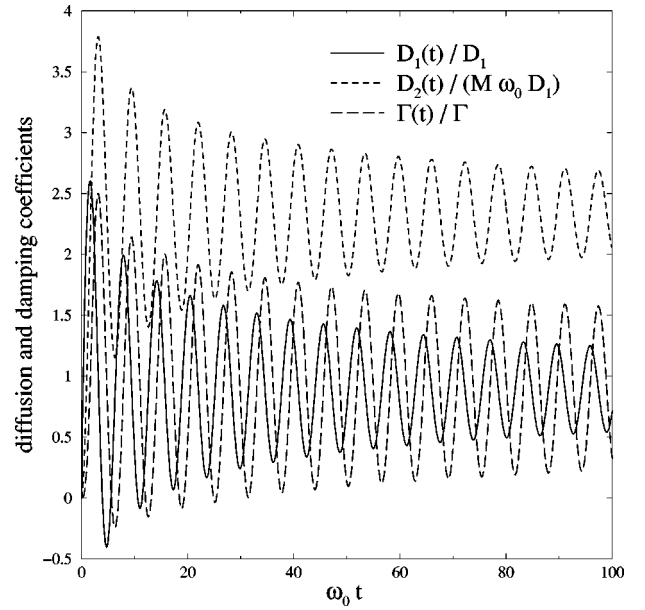


FIG. 2. Diffusion and damping coefficients for zero temperature $T=0$ as a function of time in the high-transmission limit $\omega_0/\Omega = 10^4 \gg 1$. Here $D_1 = \hbar^2 \Omega^2 \ln(\omega_0/\Omega) / 2\pi M^2 \omega_0$ and $\Gamma = \hbar \Omega^2 \ln(\omega_0/\Omega) / 2\pi M$ are the asymptotic limits of $D_1(t)$ and $\Gamma(t)$.

oscillations. Hence Eq. (31) also holds in this case, although the rate of entropy increase is much smaller than in the perfectly reflecting limit.

IV. DECOHERENCE VERSUS DAMPING

In this section, we derive a general relation between damping and decoherence time scales, starting from the fluctuation-dissipation result given by Eq. (28). As an extreme case of decoherent dynamics, we consider a superposition of two coherent states, since they correspond to the pointer states, according to the results of Sec. III. Specifically, we take at $t=0$ the even superposition state $|\psi\rangle_e = (|\alpha\rangle + |-\alpha\rangle)/\sqrt{2}$, with $\alpha = iP_0/\sqrt{2M\hbar\omega_0}$, so that the coherent states are initially along the momentum axis in phase space, and $\pm P_0$ are the average values of momentum of the components at $t=0$. We also assume that $|\alpha| \gg 1$, hence the average energy of the state components is much larger than the zero-point energy. The corresponding Wigner function is

$$W = W_m + \frac{1}{\pi\hbar} \exp\left[-\frac{q^2}{2(\Delta q_0)^2} - \frac{p^2}{2(\Delta p_0)^2}\right] \cos\left(\frac{2P_0 q}{\hbar}\right), \quad (32)$$

where $\Delta q_0 = \sqrt{\hbar/(2M\omega_0)}$ and $\Delta p_0 = \hbar/(2\Delta q_0)$ are the position and momentum uncertainties of the ground state. W_m corresponds to the statistical mixture

$$\rho_m = (|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)/2. \quad (33)$$

In phase space, W_m has two Gaussian peaks along the momentum axis at $\pm P_0$. ρ_m is a classical state in the sense that W_m , being positive defined, may be interpreted as a probability distribution in phase space. On the other hand, the

nonclassical nature of the superposition state is featured by the remaining term in Eq. (32), representing the coherent interference between the two state components, and which oscillates into negative values along the position axis.

Diffusion along position, associated to the coefficient D_1 , averages out the oscillations of the interference term at a rate $1/t_d$, to be calculated from the Fokker-Planck equation (12) [22]. According to Eq. (32), the oscillations are faster the higher the value of P_0 , so that t_d is a decreasing function of $|\alpha|$. As in Sec. III, we assume that decoherence is very slow, $1/t_d \ll \omega_0$, so that several free oscillations take place before coherence is lost. In this limit, the particle has enough time to probe the harmonic potential before diffusion takes place, and as a consequence decoherence is governed by the asymptotic value of D_1 , which is directly connected to the field fluctuations at the frequency of oscillation ω_0 , according to Eq. (27). This condition holds for most experiments, where mesoscopic superpositions are employed so as to render decoherence slow enough to be measured [3,5]. Moreover, it always applies in the case of vacuum radiation pressure ($T=0$), as shown in Sec. V. Diffusion is maximum when the state components are along the momentum axis: from Eq. (32), we find $\partial_q^2 W \approx -(2P_0/\hbar)^2 W$; and vanishes when the two wave packets reach the turning points in the harmonic potential. The average over many oscillation yields

$$\frac{1}{t_d} = -\frac{1}{2} D_1 \left(\frac{\partial_q^2 W}{W} \right)_{\max} = \frac{2P_0^2 D_1}{\hbar^2}, \quad (34)$$

that combined with the fluctuation-dissipation theorem, Eq. (28), yields the following result for the decoherence time t_d :

$$t_d = \frac{1}{4|\alpha|^2} \tanh\left(\frac{\hbar\omega_0}{2T}\right) \frac{1}{\Gamma}. \quad (35)$$

A $T=0$ (or more generally, for $T \ll \hbar\omega_0$), Eq. (35) yields $t_d = 1/(4|\alpha|^2 \Gamma)$. This result may be written in terms of the distance $\Delta P = 2P_0$ between the two components in phase space at $t=0$, or in terms of the distance $\Delta Q = \Delta P/(M\omega_0)$ at $t = \pi/2\omega_0$:

$$t_d = 4 \left(\frac{\Delta P_0}{\Delta P} \right)^2 \frac{1}{\Gamma} = 4 \left(\frac{\Delta q_0}{\Delta Q} \right)^2 \frac{1}{\Gamma}. \quad (36)$$

The interpretation of Eq. (36) is clear: decoherence is faster the more separated the state components in phase space are. Here the zero-point fluctuations define the reference of distance in phase space. At high temperatures, on the other hand, this reference is provided by the thermal de Broglie wavelength $\lambda_T = \hbar/\sqrt{2MT}$. In fact, Eq. (35) yields, for $T \gg \hbar\omega_0$,

$$t_d = \frac{\hbar\omega_0}{2T} \frac{1}{4|\alpha|^2} \frac{1}{\Gamma} = 2 \left(\frac{\lambda_T}{\Delta Q} \right)^2 \frac{1}{\Gamma}. \quad (37)$$

Equation (37) also shows that the ratio between decoherence and damping rates is larger at high temperatures by the factor $2T/(\hbar\omega_0)$.

When written in terms of distances in phase space, the results above are also valid for more general superposition states, like $(|0\rangle + |\alpha\rangle)/\sqrt{2}$. Moreover, their range of validity is not limited to the radiation pressure coupling considered here. In fact, Eqs. (36) and (37) are in perfect agreement with the results obtained in the framework of the Caldeira-Legget phenomenological model for quantum dissipation [6]. This is hardly surprising, since they rely on general properties of the correlation functions associated to the fluctuation-dissipation theorem. Equation (35), which interpolates the low- and high-temperature limits, is also discussed in Ref. [6], in the context of a two-level system. The temperature dependence for the ratio between decoherence and damping times has a simple interpretation: at $T>0$, the time scale for the relaxation of the populations is shorter than $1/\Gamma$ exactly by the factor $\tanh(\hbar\omega_0/2T)$, on account of the contribution of absorption and stimulated emission. Here this factor originates from the general relation between symmetric and antisymmetric correlation functions, Eq. (25), which is at the heart of the fluctuation-dissipation theorem.

The peculiarities of the radiation pressure model considered here are contained in the damping rate Γ . Rather than a phenomenological input parameter, it is computed from first principles, first for $T=0$ in Sec. V, and then for $T \gg \hbar\omega_0$ in Sec. VI.

V. VACUUM FIELD

At $T=0$, the spectral density is given by Eqs. (21) and (22). This result is more easily obtained from the following argument (a similar method, applied for the force correlation function, may be found in Refs. [23] and [24]). Since \mathcal{P} is quadratic in the field operators, the correlation function $C(t)$ may be calculated from the two-photon matrix elements of the momentum operator as follows:

$$C(t) = \frac{1}{2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \langle 0 | \mathcal{P}(t) | \omega_1, \omega_2 \rangle \langle \omega_1, \omega_2 | \mathcal{P} | 0 \rangle. \quad (38)$$

We have

$$\langle 0 | \mathcal{P}(t) | \omega_1, \omega_2 \rangle = \exp[-i(\omega_1 + \omega_2)t] \langle 0 | \mathcal{P}(0) | \omega_1, \omega_2 \rangle$$

since only the annihilation operators contribute, and hence

$$C[\omega] = \pi \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 |\langle 0 | \mathcal{P}(0) | \omega_1, \omega_2 \rangle|^2 \times \delta(\omega - \omega_1 - \omega_2). \quad (39)$$

Thus, at $T=0$ the fluctuations at the (positive) frequency ω , originate from two-photon states $|\omega_1, \omega_2\rangle$ such that $\omega_1 + \omega_2 = \omega$. In the dynamical Casimir effect, the oscillation at the mechanical frequency ω_0 gives rise to the emission of pairs of photons of frequencies ω_1 and ω_2 , such that $\omega_0 = \omega_1 + \omega_2$. On the other hand, according to Eq. (26), Γ originates from the fluctuations at frequency ω_0 , and hence

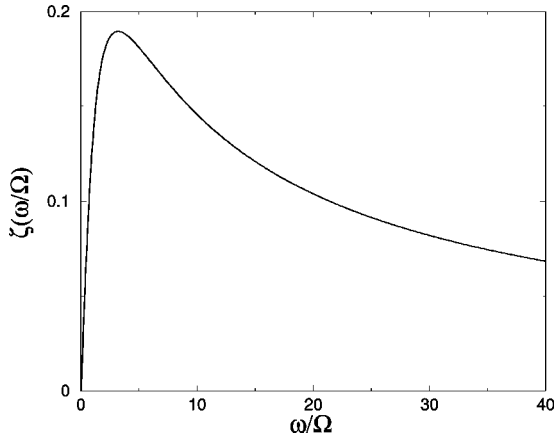


FIG. 3. Spectral density for zero temperature.

$$\Gamma = \frac{\pi}{4} \frac{\omega_0}{M\hbar} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 |\langle \omega_1, \omega_2 | \mathcal{P} | 0 \rangle|^2 \delta(\omega_0 - \omega_1 - \omega_2), \quad (40)$$

rendering explicit the connection between damping and the photon emission effect. In the Appendix, we present an alternative derivation of Eq. (40), starting from the two-photon emission probabilities and making use of energy conservation.

Equation (39) also shows that $C[\omega]$ vanishes for negative frequencies, and as a consequence, $\sigma[\omega] = \epsilon(\omega)\xi[\omega]$ in agreement with Eq. (25). Finally, the result of Eq. (21) follows from Eq. (39) by using again Eq. (17) [16]. In Fig. 3, we plot $\zeta(\omega/\Omega)$ as a function of its argument. According to Eq. (21), the transparency frequency Ω defines a scale for the behavior of the spectrum of fluctuations in vacuum. For $\omega \ll \Omega$, the spectrum is linear: $\zeta(\omega/\Omega) \approx \omega/(6\Omega)$, and goes to zero slowly, as $\ln(\omega/\Omega)/(\omega/\Omega)$, for $\omega \gg \Omega$, due to the high-frequency transparency of the mirror.

The damping coefficient at zero temperature is obtained from Eqs. (21) and (26), or alternatively from Eq. (40):

$$\Gamma = \frac{\hbar\Omega\omega_0}{2\pi M} \zeta\left(\frac{\omega_0}{\Omega}\right). \quad (41)$$

In the perfectly reflecting limit, $\omega_0 \ll \Omega$, Eq. (41) yields

$$\Gamma = \frac{\hbar\omega_0}{12\pi M} \omega_0. \quad (42)$$

Thus, the damping induced by the Casimir effect is a small perturbation of the free harmonic oscillations. The ratio between the zero-point energy and the rest energy appearing in Eq. (42) is also of the order of the recoil velocity of the mirror divided by c , which is, as explained in Sec. II, the small parameter of the perturbation approach leading to the master equation (11). For larger values of ω_0/Ω , the damping as given by Eq. (41) is still smaller, since vacuum frequencies of the order of ω_0 are not well reflected by the mirror in this case.

Equation (42) is directly connected to the well-known formula for the dissipative Casimir force on a single perfect moving mirror [11], $F = \hbar x'''/(6\pi)$, for the equation of motion then reads [28]

$$x'' = -\omega_0^2 x + \frac{\hbar x'''}{6\pi M}, \quad (43)$$

whose solution when $\hbar\omega_0/M \ll 1$ is

$$x = x_0 e^{-i\omega_0 t} \exp\left(-\frac{\hbar\omega_0^2}{12\pi M} t\right).$$

The decoherence time scale at $T=0$ in the perfectly reflecting limit is derived from Eqs. (36) and (42):

$$t_d = \frac{3}{v^2} \frac{2\pi}{\omega_0}, \quad (44)$$

where $v = P_0/M$ is the initial velocity of the wavepackets. Being of the order of $(v/c)^2$, the decoherence rate is very small at $T=0$ (or, at any rate, in the nonrelativistic limit considered here). Since $\omega_0 t_d \gg 1$, decoherence is the cumulative effect of several free oscillations in the harmonic well, which justifies the approach employed in the derivation of Eq. (34) and the use of the asymptotic value for $D_1(t)$.

In order to further understand how the dynamical Casimir effect engenders decoherence, we present, in the Appendix, an alternative approach, where we follow the evolution of the complete oscillator-plus-field quantum state. It shows that the superposition state decoheres because the two wavepacket components oscillating out-of-phase yield amplitudes for emission of photon pairs with opposite signs. As a consequence, an entangled mirror-plus-field state is developed, given by

$$\begin{aligned} |\Psi\rangle_{\Delta t} = & B(\Delta t) |\psi\rangle_e \otimes |0\rangle \\ & + \frac{1}{2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 b(\omega_1, \omega_2; \Delta t) |\psi\rangle_o \otimes |\omega_1, \omega_2\rangle, \end{aligned} \quad (45)$$

where $|\psi\rangle_o = (|\alpha\rangle - |-\alpha\rangle)/\sqrt{2}$ is the odd superposition state, $b(\omega_1, \omega_2; \Delta t)$ is the amplitude for emission of a photon pair with frequencies ω_1 and ω_2 during Δt (the explicit expressions are given in the Appendix), and B is the amplitude for persistence in the vacuum state:

$$|B(\Delta t)|^2 = 1 - \frac{1}{2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 |b(\omega_1, \omega_2, \Delta t)|^2. \quad (46)$$

The density operators of the odd and even superposition states differ by the sign of the interference term, $\rho_{\text{int}} = \rho - \rho_m$ [ρ_m is defined in Eq. (33)]. Accordingly, when computing the reduced density matrix of the mirror, $\rho(\Delta t) = \text{tr}_{\text{field}}(|\Psi\rangle_{\Delta t} \langle \Psi|)$, the contribution of the two-photon states in Eq. (45) reduces the coherence of the state. With the help of Eq. (46), we find

$$\begin{aligned}\Delta\rho_{\text{int}} &\equiv \rho_{\text{int}}(\Delta t) - \rho_{\text{int}}(0) \\ &= -\frac{1}{2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 |b(\omega_1, \omega_2; \Delta t)|^2 \rho_{\text{int}}(0).\end{aligned}\quad (47)$$

The two-photon probabilities are proportional to Δt and connected to the damping rate Γ as discussed in the Appendix. Hence Eq. (47) yields

$$\frac{d\rho_{\text{int}}}{dt} \approx \frac{\Delta\rho_{\text{int}}}{\Delta t} = -\frac{\rho_{\text{int}}}{t_d}, \quad (48)$$

with t_d given by Eq. (36).

In this derivation, the expression for the emission amplitudes *per se* are not of any relevance—only its connection with the damping rate Γ is important. This connection is based on the principle of energy conservation: the energy of the oscillator is damped at the rate at which energy is radiated. Since this argument also holds for the real 3+1 electromagnetic field, we may extend our results by replacing the 3D result for Γ into Eq. (36). The dissipative dynamical Casimir force on an oscillating (frequency ω_0) perfectly reflecting sphere was obtained in Ref. [23]. Usually, the sphere is very small when compared with the wavelength of the relevant vacuum fluctuations, which are of the order of $2\pi/\omega_0$. When $\omega_0 R \ll 1$, the force on the sphere of radius R is given by

$$F = \frac{-\hbar R^6}{648\pi} x^{(9)}, \quad (49)$$

where $x^{(9)}$ is the ninth time derivative of the position of the sphere. Following again the method of Eq. (43), we calculate the damping rate Γ from the equation of motion. We find

$$\Gamma = \frac{1}{1296\pi} \frac{\hbar \omega_0^8 R^6}{M}, \quad (50)$$

showing that the coupling with the vacuum field is reduced, as compared with the 1D case, by the (very small) factor $(\omega_0 R)^6$. This reduction factor accounts for the inefficient coupling of the small particle, which scatters field modes of very long wavelengths. Using Eq. (36), we find that the decoherence time increases by the same factor:

$$t_d = \frac{324}{v^2} \frac{1}{(\omega_0 R)^6} \frac{2\pi}{\omega_0}, \quad (51)$$

hence decoherence through radiation pressure is a tiny effect at $T=0$. At finite temperatures, however, the effect may be significant, as discussed in the next section.

VI. HIGH-TEMPERATURE LIMIT

In this section, we compute the damping and decoherence rates when $T \gg \hbar \omega_0$. In this limit, vacuum fluctuations are negligible when compared with thermal fluctuations, and the dominant contribution in Eq. (20) comes from ξ^T , which is

given by Eq. (23). When the temperature is also higher than the cutoff energy $\hbar \Omega$, all relevant frequencies in Eq. (23), which are smaller or of the order of Ω , are much smaller than T/\hbar . Then, we may take the approximation $n_{\omega'} \approx T/(\hbar \omega')$, yielding

$$\xi^T[\omega_0] = 2 \frac{\hbar \Omega T}{\omega_0}. \quad (52)$$

Replacing Eq. (52) into Eq. (26) yields

$$\Gamma = \frac{\Omega T}{2M}, \quad (53)$$

in agreement with the result for the viscous radiation pressure force obtained in Ref. [26]: $F = -\Omega T \dot{q}(t)$.

From a practical point-of-view, the opposite limit, $\hbar \omega_0 \ll T \ll \hbar \Omega$ is more interesting for particles that scatter visible light ($\Omega \sim 10^{16}$ Hz). In this case, the corresponding reflectivity amplitude $R(\omega)$ is approximately constant for the field modes whose frequencies are smaller or of the order of T/\hbar . As a consequence, we may neglect the Lorentzian falloff in Eq. (23). Moreover, we replace the thermal photon number $n_{\omega' - \omega_0}$ in Eq. (24) by

$$n_{\omega' - \omega_0} \approx [\exp(\hbar \omega'/T)(1 - \hbar \omega_0/T) - 1]^{-1}. \quad (54)$$

Neglecting second- and higher-order terms in $\hbar \omega_0/T$, we find

$$G(\omega_0, \omega') = \frac{\omega' e^{\hbar \omega'/T}}{(\hbar \omega'/T - 1)^2} \frac{\hbar \omega_0}{T}. \quad (55)$$

From Eqs. (23) and (55) we find

$$\xi^T[\omega_0] = \frac{4\pi}{3} \frac{T^2}{\omega_0}, \quad (56)$$

and then

$$\Gamma = \frac{\pi}{3} \frac{T^2}{M \hbar}, \quad (57)$$

which is also in agreement with Ref. [26]. It corresponds to the high-temperature, perfectly reflecting limit. Here T plays the role of frequency cutoff instead of Ω , so that the resulting damping rate is independent of the latter.

The dissipative force in the high-temperature limit may be interpreted as the effect of Doppler shift of the reflected thermal photons [26]. For a photon of frequency ω , the frequency shift is $\Delta\omega = \pm 2\omega \dot{q}$, the plus and minus signs applying for counter and co-propagating cases. Hence the motion gives rise to an unbalance between the radiation pressure exerted on each side of the mirror, corresponding to a momentum transfer $\Delta P = 2\Delta E \dot{q}$, where ΔE is the reflected energy during a time interval Δt . In terms of the density of modes $g(\omega)$, we have

$$\Delta E = \int_0^\infty d\omega |R(\omega)|^2 g(\omega) n_\omega \hbar \omega, \quad (58)$$

where $|R(\omega)|^2$ represents the mirror reflectivity [the reflection amplitude R is given by Eq. (4)]. From ΔE , the friction force is obtained through

$$F = -2 \frac{\Delta E}{\Delta t} \dot{q}. \quad (59)$$

In the 1D case, the density of modes is frequency independent: $g(\omega)d\omega = (L/\pi)d\omega$, where $L = \Delta t$ is the length of the quantization box. When replaced into Eq. (58), this result leads, with the help of Eq. (59), to expressions for the force in agreement with our results for Γ , except for a factor of 2 when $\hbar\omega_0 \ll T \ll \hbar\Omega$ [29]. In the 3D case, on the other hand, we have

$$g(\omega)d\omega = \frac{V}{\pi^2} \omega^2 d\omega, \quad (60)$$

where $V = A\Delta t$ is the quantization volume, A being the surface of the mirror (in this case, for simplicity, we assume a flat rather than spherical mirror). In the limit $\hbar\omega_0 \ll T \ll \hbar\Omega$, Eqs. (58) and (60) yield

$$\frac{\Delta E}{\Delta t} = \frac{\hbar A}{\pi^2} \int_0^\infty d\omega \frac{\omega^3}{\exp(\hbar\omega/T) - 1} = \frac{\pi^2}{15} \frac{AT^4}{\hbar^3}. \quad (61)$$

As expected, the reflected power features the T^4 dependence of the Stefan-Boltzmann law, since it is proportional to the total blackbody radiation energy in this limit. The friction force is found by replacing Eq. (61) into Eq. (59). The resulting damping coefficient is given by

$$\Gamma = \frac{\pi^2}{15} \frac{A}{\hbar^3} \frac{T^4}{M}. \quad (62)$$

Since we have neglected diffraction at the borders of the mirror, this result only applies when the mirror is much larger than the thermal photon wavelength $\lambda_{\text{th}} = 2\pi\hbar/T$.

The decoherence time is then found by replacing Eq. (62) into Eq. (37), which connects damping and decoherence in the high-temperature limit (we reintroduce the speed of light c in order to allow an evaluation of the orders of magnitude):

$$t_d = \frac{15}{32\pi^7} \frac{\lambda_{\text{th}}^5}{cA\Delta Q^2}. \quad (63)$$

As a numerical example, we take $T = 50$ K, which gives $\lambda_{\text{th}} = 2.9 \times 10^{-4}$ m, and $A = 1$ mm². In this case, diffraction is negligible, and Eq. (63) yields $t_d [\text{s}] = 1.0 \times 10^{-24} / (\Delta Q^2 [\text{m}^2])$, showing that decoherence is very fast even when the distance between the wave packets is, for instance, in the nanometer range—in this case the decoherence time is of the order of a microsecond. Since t_d scales as $1/T^5$, it is still shorter, by a factor $\approx 8 \times 10^3$, at room temperature. Note, however, that Eq. (63) only applies when $\omega_0 t_d \gg 1$, the

basic assumption that allowed us to relate decoherence and damping time scales with the help of the fluctuation-dissipation theorem [30].

VII. CONCLUSIONS

As in the Caldeira-Leggett model [20], coherent states are the most robust when the radiation pressure coupling with the quantum field is considered. This is amazingly in line with their well-known status of “quasiclassical” states, i.e., the closest possible representation of a classical oscillation in a harmonic potential well. Superpositions of coherent states decay into a mixture at a rate proportional to the damping rate and to the squared distance in phase space. The ratio between decoherence and damping rates is a simple hyperbolic increasing function of temperature, which interpolates the zero and high-temperature limits. It originates from the general relation between symmetric and antisymmetric correlation functions, associated to the fluctuation-dissipation theorem. Thus, the particular nature of the model for the coupling with the reservoir seems to be immaterial, as far as the connection between damping and decoherence is concerned. Note, however, that the validity of this result is limited by the assumption that decoherence is slow compared to the free oscillations.

We have shown that the radiation pressure exerted by thermal photons is a very efficient source of decoherence, although the corresponding energy damping effect, associated to the Doppler frequency shift of the reflected photons, is usually negligible. At $T = 0$, the energy damping is associated to the emission of photon pairs (dynamical Casimir effect). The dominant contribution comes from vacuum fluctuations corresponding to wavelengths of the order of $2\pi c/\omega_0$, which is usually much greater than the size of the oscillator. As a consequence, the radiation pressure coupling is inefficient, and both damping and decoherence rates become very small. It is however remarkable, from a theoretical point of view, that the mere inclusion of an unavoidable, intrinsically quantum effect, is sufficient (in principle) to engender decoherence, and by that means restoring, although in a very long-time scale, the classical world.

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APPENDIX: ENTANGLEMENT WITH TWO-PHOTON STATES

In this Appendix, we present an alternative, simpler derivation of the decoherence time scale at $T = 0$, which shows more clearly how the dynamical Casimir effect modifies the quantum phase of a superposition state and engenders decoherence. Instead of tracing over the field, we follow its evolution during many periods of free oscillation. We first take, at $t = 0$, the mirror-plus-field state $|\alpha\rangle \otimes |0\rangle$ ($|0\rangle$ represents

the vacuum field state), where $|\alpha\rangle$ is a coherent state of large amplitude: $|\alpha| \gg 1$. We take $\alpha = i\dot{q}(0)\sqrt{M/2\hbar\omega_0}$, so that $|\alpha\rangle$ is a “semiclassical” state associated to a minimum uncertainty wave packet whose initial velocity is $\dot{q}(0)$. We have shown in Sec. IV that the action of the vacuum radiation pressure on the motion of the mirror is a very small perturbation (weak-coupling limit). Thus the time evolution may be computed from a simple semiclassical model, in which the field evolution is obtained assuming the classical *prescribed* motion

$$\dot{q}(t) = \dot{q}(0)\cos(\omega_0 t), \quad (\text{A1})$$

where $q(t)$ is the position of the mirror at time t . The dynamical Casimir effect is described by the interaction Hamiltonian [see Ref. [16], and compare with the first term in Eq. (5)]

$$H_{\text{int}} = -\dot{q}(t)\mathcal{P}. \quad (\text{A2})$$

The amplitude $b(\Delta t)$ for the creation of photon pairs corresponding to frequencies ω_1 and ω_2 at time Δt is given by

$$b(\omega_1, \omega_2; \Delta t) = \frac{i}{\hbar} \langle \omega_1, \omega_2 | \mathcal{P} | 0 \rangle \int_0^{\Delta t} dt' e^{i(\omega_1 + \omega_2)t'} \dot{q}(t'). \quad (\text{A3})$$

According to Eq. (A3), the amplitude depends on the *sign* of \dot{q} , which is very important to the discussion of decoherence.

Replacing Eq. (A1) into (A3), we find for the two-photon probabilities

$$|b(\omega_1, \omega_2; \Delta t)|^2 \approx \frac{1}{\hbar^2} |\langle 0 | \mathcal{P} | \omega_1, \omega_2 \rangle|^2 \dot{q}(0)^2 \times \frac{\sin^2[(\omega_1 + \omega_2 - \omega_0)\Delta t/2]}{(\omega_1 + \omega_2 - \omega_0)^2}. \quad (\text{A4})$$

For $\omega_0 \Delta t \gg 1$, the right-hand side of Eq. (A4) is sharply peaked around $\omega_1 + \omega_2 = \omega_0$. Thus, for large times energy is well defined, in agreement with the time-energy uncertainty relation. In this limit, Eq. (A4) yields

$$|b(\omega_1, \omega_2; \Delta t)|^2 \approx \frac{\pi}{2\hbar^2} |\langle 0 | \mathcal{P} | \omega_1, \omega_2 \rangle|^2 \dot{q}(0)^2 \Delta t \times \delta(\omega_1 + \omega_2 - \omega_0). \quad (\text{A5})$$

Since the source of the radiated energy is the motion of mirror, one may expect that the two-photon probabilities are related to the amplitude decay rate Γ . The radiated energy during Δt is

$$\Delta E = \frac{1}{2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 |b(\omega_1, \omega_2; \Delta t)|^2 \hbar(\omega_1 + \omega_2), \quad (\text{A6})$$

which according to Eq. (A5) is proportional to the time interval Δt . The energy of the mirror decays as $dE_M/dt = -2\Gamma E_M$, where $E_M = M\dot{q}(0)^2/2$. Hence, from energy conservation we have

$$\Gamma = \frac{1}{M\dot{q}(0)^2} \frac{\Delta E}{\Delta t},$$

leading, with the help of Eqs. (A5) and (A6), to the representation given by Eq. (40).

To analyze the effect of decoherence, we take the field to be initially in the “even” superposition state $|\psi\rangle_e = (|\alpha\rangle + |-\alpha\rangle)/\sqrt{2}$, so that the mirror-plus-field state at $t=0$ is

$$|\Psi\rangle_0 = |\psi\rangle_e \otimes |0\rangle.$$

By linearity, its time evolution is obtained from the two-photon amplitudes given by Eq. (A3):

$$|\Psi\rangle_{\Delta t} = (|\alpha\rangle \otimes |\phi^+\rangle_{\Delta t} + |-\alpha\rangle \otimes |\phi^-\rangle_{\Delta t})/\sqrt{2}, \quad (\text{A7})$$

where

$$|\phi^\pm\rangle_{\Delta t} = B(\Delta t)|0\rangle \pm \frac{1}{2} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 b(\omega_1, \omega_2; \Delta t) |\omega_1, \omega_2\rangle. \quad (\text{A8})$$

The already noted sensitivity of the two-photon amplitudes to the phase of the motion of the mirror, which is explicit through the “minus” sign for $|\phi^-$ in Eq. (A8), generates entanglement between mirror and field. This is discussed in Sec. V, whose starting point is Eq. (45), which is derived by replacing Eq. (A8) into Eq. (A7).

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